

# Signals, systems, acoustics and the ear

## Lab 3: Harmonic Synthesis

### **Introduction**

Any waveform in the real world can be constructed by adding together sine waves of the appropriate amplitudes, frequencies and phases. A *spectrum* is the specification of the sinusoids that are needed to construct a given wave. You can think of it as a list of ingredients for a very simple recipe. For a periodic waveform, the spectrum is relatively simple, in that it can only have sinusoidal components at the fundamental frequency of the original wave, and whole number multiples thereof. These components are called *harmonics*. In this laboratory, we will build periodic waveforms from their constituent sinewaves: *harmonic synthesis*. *Harmonic analysis* occurs when we are given a particular periodic wave, and determine what constituent sinusoids are necessary to build it up (analysis='breaking down'; synthesis='building up').

### **Learning Objectives**

- To understand the terms: fundamental frequency, harmonic, amplitude, phase, and spectrum.
- To be able to interpret a spectrum.

### **Apparatus**

You will use a computer program (*Esynth*), freely available on the Web, for harmonic synthesis. *Esynth* allows you to specify an amplitude spectrum either by using the mouse in a graphical mode, or by editing a table which specifies the component frequencies, amplitudes and phases (press the *EDIT* button on the toolbar to bring up the control panel). You will see the resulting waveform and be able to play it to see how it sounds. You will need to zoom in on the waveform to see the shape of the waves. (Playing the sound from the control panel also allows you to hear the whole waveform while being zoomed in so you can see separate cycles.)

**Important!** Note that specifying  $0^\circ$  phase in this program results in a wave that starts at 0 and then goes up (normally known as *sine phase*), so that  $90^\circ$  phase results in a wave that starts at 1 and then goes down (normally known as *cosine phase*).

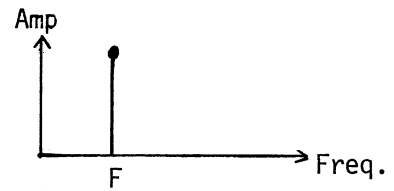
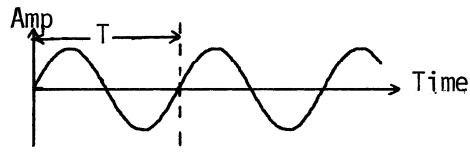
<http://www.phon.ucl.ac.uk/resource/sfs/esynth.htm>

### **Observations**

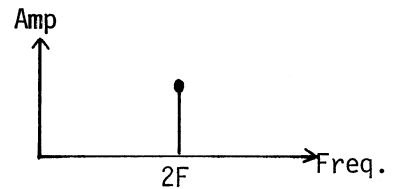
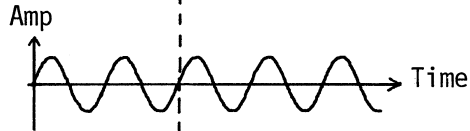
For observations 1-5 and 7, ensure the program is in its 'harmonic' mode (Options menu: Harmonise ticked, Fundamental frequency = 200 Hz). All sinewave frequencies will be forced to the nearest multiple of 200 Hz.

1. Single sine waves. Use the mouse to move a single harmonic to different frequencies, amplitudes and phases. Note the effect of changing these three parameters on the waveform and the sound.
2. Simple wave shapes. Use the program to recreate the waveforms on the next page of this handout composed of the first, second and third harmonic in different combinations (1+2, 1+3, 2+3, 1+2+3). The 2<sup>nd</sup> and 3<sup>rd</sup> harmonics are equal in amplitude, and 2/3rds of the size of the first harmonic. In each case measure the fundamental period. What effect does changing the phase of the first harmonic have on (i) the waveshape, (ii) the quality of the sound? What can you infer from this?

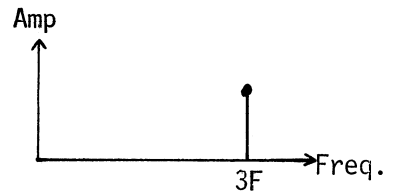
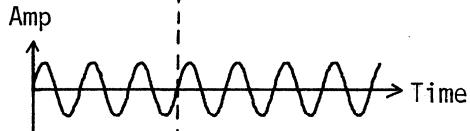
Waveform A  
 Frequency =  $F$   
 Period =  $T$   
 Amplitude = 0.9



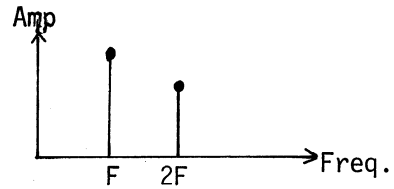
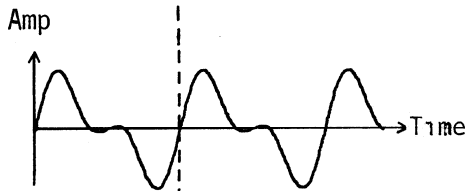
Waveform B  
 Frequency =  $2F$   
 Period =  $T/2$   
 Amplitude = 0.6



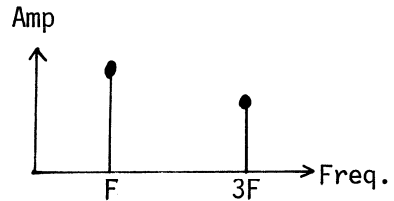
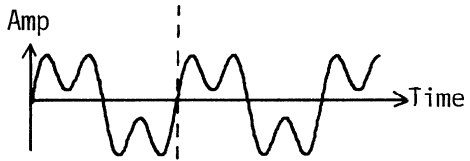
Waveform C  
 Frequency =  $3F$   
 Period =  $T/3$   
 Amplitude = 0.6



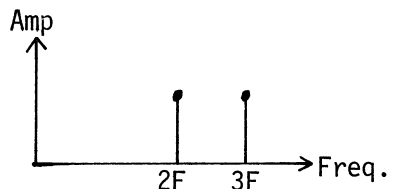
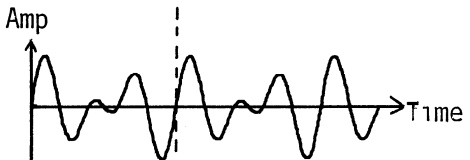
Waveform A+B  
 Period =  $T$



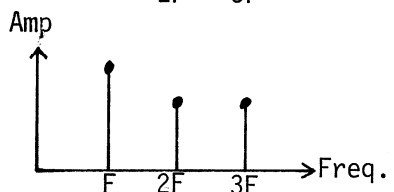
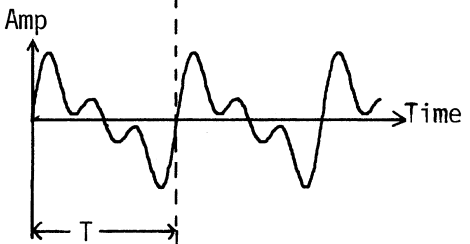
Waveform A+C  
 Period =  $T$



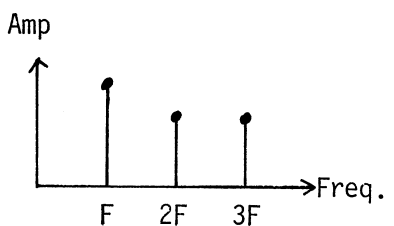
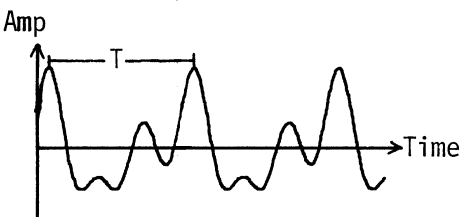
Waveform B+C  
 Period =  $T$



Waveform A+B+C  
 Period =  $T$



Waveform A+B+C  
 with A shifted by  $+90^\circ$   
 Period =  $T$



3. Designer waveshapes: A sawtooth wave. Chapter 7 in *Signals & Systems for Speech & Hearing* describes the construction of a sawtooth wave, which contains all harmonics in sine phase, but whose amplitude decreases with increasing frequency. The amplitude of the  $n$ th harmonic should be set to  $1/n$ . Complete the table below before you start:

harmonic frequency	harmonic number	amplitude (fraction)	amplitude (decimal)	amplitude (dB re 1)	Phase (degrees)
200	1	1	1	0	0
400	2	1/2	0.5	-6	0
600	3	1/3	0.333		0
800	4	1/4			0
1000	5	1/5			0
1200	6	1/6			0
1400	7	1/7			0
1600	8	1/8			0

Build up the sawtooth wave one component at a time, both looking at the waveform and listening to it. If you want to use a decibel scale, instead of a linear scale, select *Decibel scale* under the *Options* menu. Note that every doubling in frequency (a 1 octave change) results in a 6 dB attenuation of the harmonic. We thus say that the spectrum rolls off at -6 dB/octave.

Recall that the formula for decibels is just

$$amplitude(dB) = 20 \log_{10} \left( \frac{amplitude}{reference} \right)$$

*Esynth* always uses a reference of 1 for this calculation.

4. The case of the missing fundamental. From the waveform of the sawtooth generated above, measure the period of the wave using the cursors (set by the left and right mouse bottom, with the time between them and the corresponding frequency displayed in the lower right-hand corner of the display window). What is the period and frequency? Now, change the fundamental component of the sawtooth (the one at 200 Hz) to an amplitude of zero. Measure the period and frequency again. Has it changed? You may find it odd that a periodic wave with a fundamental frequency of 200 Hz has no energy at 200 Hz. Think about this! Try setting the first 3 or 4 harmonics of the sawtooth to zero as well. Does the periodicity (and hence the fundamental frequency) change?
5. Vowel-like waveforms. Try to build waves which sound like vowels. You can get an idea of where to put harmonics by analysing a steady vowel of your own. The toolbar button marked with the red circle enables recording. Record a sustained 'ee' or 'ah' on a monotone. It would be best to utter the vowel continuously, and then start and stop recording so you get a continuous waveform within the time window of the display. You can then analyse the wave by choosing *Signal: Fourier Analysis (Find Fx)*. You will see a simplified harmonic spectrum of the original vowel in the bottom window, and the waveform of the synthesised vowel in the middle window (which can be played by pressing the blue arrow). You can *EDIT* this table of values if you like. Sometimes the program makes a mistake in finding the fundamental frequency (Fx). Check this by making an independent measurement of the Fx. Zoom in to a few cycles of the input waveform, and place the left cursor (left mouse button) at the start of a cycle, and the right cursor (right mouse button) at the end of the cycle. The time between these, and the

calculated frequency can be found at the bottom right of the Esynth window (e.g.,  $T=0.0058\text{s}$   $F=172\text{ Hz}$ ). Check that this  $F_x$  value is the same as the frequency of the first harmonic on the output spectrum. If not, select a different section of your vowel and retry.

6. Beats. Switch the program to anharmonic mode (Options: *Harmonise* not ticked). Use the *EDIT* control panel to create two sinewaves close together in frequency so that you hear not two sounds but one sound changing in amplitude. Zoom in on the component waves (select *Output Components* on the *View* menu item), and scroll through them to see how sometimes the two components are pretty much in phase, and so add *constructively*, but at other times, are pretty much out of phase, so add *destructively*. This perceptual phenomenon is known as *beats* and it is related to the capability of the ear to separate out the individual frequency components of a sound. Estimate the period of the beating waveform. How do you think it relates to the frequencies of the components involved?
7. Other designer wave shapes: Use the program (back in 'harmonic' mode) to create a square and triangular wave as seen on page 5 of this handout. For a square wave, you will only need to use the odd numbered harmonics, and, as for the sawtooth, the amplitude of the  $n$ th harmonic should be set to  $1/n$ . You may like to complete the table below before you start:

harmonic	amplitude	amplitude (dB)	phase
1	1	0	0
3	1/3	-9.5	0
5	1/5		0
7	1/7		
9			

For a triangular wave, again only use the odd numbered harmonics, but with amplitudes  $1/n^2$  for the  $n$ th harmonic, alternately changing in phase between  $0^\circ$  and  $180^\circ$ .

harmonic	amplitude	amplitude (dB)	phase
1	1	0	0
3	1/9		180
5	1/25		0
7	1/49		
9			

Why does a square wave sound different to a triangular wave of the same amplitude and repetition frequency?

### **Things to think about**

1. Put arguments against the claim: "Surely, some complex periodic sounds will need to be synthesised from an infinite number of harmonics".
2. How would the spectrum of a sawtooth wave change with increasing amplitude?
3. How would the spectrum of a sawtooth wave change with increasing fundamental frequency?